AP® Calculus BC
2004 Sample Student Responses
Form B

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A₁

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$$\text{speed} = \sqrt{(x'(0))^2 + (y'(0))^2}$$

$$\frac{dx}{dt}(0) = \sqrt{0^4+9} = \sqrt{9} = 3 \qquad \frac{dy}{dt}(0) = 2e^0 + 5e^0 = 2+5=7$$

$$\text{speed} = \sqrt{3^2 + 7^2} = \sqrt{58} = 7.616 \text{ (3d.p)}$$

$$\text{acceleration: } \frac{d^2x}{dt^2} = \frac{1}{2\sqrt{t^4+9}}(4t^3) = \frac{2t^3}{\sqrt{t^4+9}}.$$

$$\frac{d^2y}{dt^2} = 2e^t - 5e^{-t}.$$

$$\left(\frac{2t^3}{\sqrt{t^4+9}}, 2e^t - 5e^{-t}\right) \Rightarrow \begin{matrix}\text{acceleration}\\ \text{at } t=0\end{matrix} (0, -3)$$

Work for problem 1(b)

$$m_T = \text{slope} = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2e^t + 5e^{-t}}{\sqrt{t^4+9}} \quad \text{at } t=0 \quad \boxed{\frac{dy}{dx} = \frac{7}{3}}$$

$$y - y_0 = m_T(x - x_0) \Rightarrow y - 1 = \frac{7}{3}(x-4) \Rightarrow y - 1 = \frac{7}{3}x - \frac{28}{3}$$

$$\Rightarrow y = \frac{7}{3}x - \frac{25}{3} \Rightarrow \boxed{3y = 7x - 25}$$

Continue problem 1 on page 5.

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A_2

Work for problem 1(c)

$$d = \int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^3 \sqrt{t^4+9 + (2e^t+5e^{-t})^2}\, dt$$

$$= \int_0^3 \sqrt{t^4+9 + 4e^{2t} + 20 + 25e^{-2t}}\, dt$$

$$= \int_0^3 \sqrt{t^4 + 4e^{2t} + 25e^{-2t} + 29}\, dt$$

$$= 45.227$$

Work for problem 1(d)

$$\frac{dx}{dt} = \sqrt{t^4+9} \Rightarrow dx = \sqrt{t^4+9}\, dt. \Rightarrow x = \int_0^3 \sqrt{t^4+9}\, dt.$$

since $\frac{dx}{dt} \neq 0$ & $\frac{dy}{dt} \neq 0$. ⇒ particle does not stop
→ moves in a straight line.

x-coordinate at $t=3$ $= x(0) + \int_0^3 \sqrt{t^4+9}\, dt$

$$= 4 + 13.931 = 17.931.$$

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D1

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

Speed = magnitude = $\sqrt{\frac{dx}{dt}^2 + \frac{dy}{dt}^2}$

acceleration vector

$\left(\frac{1(4t^3)}{2(t^4+9)^{\frac{1}{2}}}, \; 2e^{t} - 5e^{-t} \right)$

$= \sqrt{(\sqrt{t^4 9})^2 + (2e^{t} + 5e^{-t})^2}$

$t = 0 \rightarrow$ speed $= \sqrt{(\sqrt{9})^2 + (2e^{t} + 5e^{-t})^2}$

49

$= \sqrt{58}$

Work for problem 1(b)

$t = 0$

slope $= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2+5}{3} = \frac{7}{3}$

point of tangency: (4, 1)

$y - 1 = \frac{7}{3}(x - 4$

$y \cancel{2} = \frac{7}{3}x - \frac{28}{3} + 1$

$\boxed{y = \frac{7}{3}x - \frac{31}{3}}$

Continue problem 1 on page 5.

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D₂

Work for problem 1(c)

Total dist = length of curve

$$L = \int_0^3 \sqrt{\left(\sqrt{t^4+9}\right)^2 + \left(2e^{t}+5e^{-t}\right)^2}$$

$$= 45.227$$

Work for problem 1(d)

$$x(0) = \int_0^0 (t^4+9)^{\frac{1}{2}} + C$$

~~$= \int (u)^{\frac{1}{2}}$~~

$t = 0$

$$x(0) = 0 + C = 4, \quad \boxed{C=4}$$

$$x(3) = \int_0^3 (t^4+9)^{\frac{1}{2}} + 4 =$$

$$= 13.9307 + 4 = 17.931$$

(x coordinate)

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A1

Work for problem 2(a)

$$T(x) = f(2) + f'(2)(x-2) + \frac{f''(2)(x-2)^2}{2} + \frac{f'''(2)(x-2)^3}{3!}$$

$$= 7 + 0 \quad -9(x-2)^2 - 3(x-2)^3$$

$$\boxed{f(2) = 7}$$

$$\frac{f''(2)\cancel{(x-2)^2}}{2} = -9\cancel{(x-2)^2}$$

$$\boxed{f''(2) = -18}$$

Work for problem 2(b)

Yes there is. to have this Tailor polynomial, $f'(2) = 0$ must be true

$f''(2) = -18$, which is negative, meaning curve is concave down

if curve is concave down, and 1st derivative is 0, it is a relative max

$\boxed{f(2) \text{ is a relative maximum}}$

Continue problem 2 on page 7.

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Work for problem 2(c)

$T(0) = 7 - 9(0-2)^2 - 3(0-2)^3 = -5$

$f(0) \approx T(0) = -5$

No there isn't, because we do not know any of the derivatives at 0, so nothing can be determined.

Work for problem 2(d)

$$\frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1} = \text{max error}$$

$$\frac{f^{(4)}(c)}{(4)!}(x-2)^4 \leq \frac{6}{4!}(x-2)^4 = \frac{(x-2)^4}{4} = \text{max error}$$

$$\frac{(0-2)^4}{4} = 4 = \text{max error}$$

$-5 \pm 4 < 0$, all values within error range is still negative, so $f(0)$ is negative

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Work for problem 2(a)

$$T(x) = f(2) + f'(2)(x-2) + \frac{f''(2)(x-2)^2}{2!} + \frac{f'''(2)(x-2)^3}{2!}$$

$$T(x) = 7 + 0 - 9(x-2)^2 - 3(x-2)^3$$

$$\boxed{f(2) = 7}$$

$$\frac{f''(2)\cancel{(x-2)^2}}{2!} = -9\cancel{(x-2)^2}$$

$$f''(2) = -9(2)$$

$$\boxed{f''(2) = -18}$$

Work for problem 2(b)

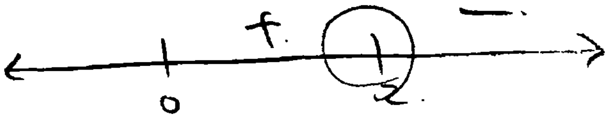
$$T(x) = 7 - 9(x-2)^2 - 3(x-2)^3$$

$$T'(x) = -18(x-2) - 9(x-2)^2 = 0$$

critical pts: → $x = 2, 0$

$T'(1) = 9$

$T'(3) = -27$



relative max.

yes there is enough information to determine whether f has a critical pt. at $x = 2$.

f(2) is a relative maximum by the 1st derivative test.

Continue problem 2 on page 7.

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C_2

Work for problem 2(c)

· $f(0) \approx T(0) = 7 - 9(0-2)^2 - 3(0-2)^3$

· $\boxed{f(0) \approx T(0) = -5}$

No you can't determine whether f has a critical point at $x=0$ because we do not know f. T(x) is a taylor polynomial about $x=2$, thus it is only an approximation. $x=0$ is too far from where T(x) is centered, thus the approximation is very inaccurate.

Work for problem 2(d)

$$\text{Lagrange error bound} = \frac{f^{(n+1)}(z)\,(x-c)^{(n+1)}}{(n+1)!}$$

$$= \frac{f^4(z)\,(x-2)^4}{4!}$$

when $x=0$.

max is 6

$$= \frac{f^4(z)(-2)^4}{4!} = \frac{6\,(-2)^4}{4!} = -4.$$

The max error bound is −4, thus f(0) lies somewhere is T(0) ± error, somewhere in −9 to −1. Thus f(0) is negative.

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B1

t (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

Work for problem 3(a)

~~=10t~~

~~Area = 10(9.2) + 10(7.5)~~

$\text{area} = 10(f(5) + f(15) + f(25) + f(35))$

$= 10(9.2 + 7 + 2.4 + 4.3)$

$\boxed{\text{area} = 229 \text{ miles}}$

$\int_0^{40} v(t)\,dt$ is the total distance traveled between $t=0$ and $t=40$ minutes

Work for problem 3(b)

$a(t) = 0$

~~between~~ [illegible]

on the intervals $[0,15]$ and $[25,30]$

the smallest number of instances the acceleration can equal zero is 2 by MVT and Rolle's Theorem

Continue problem 3 on page 9.

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B_2

Work for problem 3(c)

$f'(t) = -\frac{1}{10}\sin t/10 + 3\cdot\frac{7}{40}\cos 7t/40$

$f'(t) = -1/10 \sin t/10 + 21/40 \cos 7t/40$

$f'(23) = -1/10 \sin 23/10 + 21/40 \cos 161/40$

$f'(23) = \boxed{-.408 \text{ miles per minute}^2}$

Work for problem 3(d)

~~Average v = $\frac{f(40) - f(0)}{40-0}$~~

~~$= \frac{7.317 - 6}{40}$~~

~~= .033 miles per minute~~

$\frac{1}{40-0}\int_0^{40} 6 + \cos(t/10) + 3\sin(7t/40)\,dt$

average velocity $= \frac{1}{40}\cdot 236.65079$

$\boxed{= 5.916 \text{ miles per minute}}$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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C1

t (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

Work for problem 3(a)

$$\int_0^{40} v(t)\,dt = \frac{40-0}{4}\left[9.2 + 7 + 2.4 + 4.3\right]$$

$$= \frac{40}{4}\left[22.9\right] = 229 \text{ miles}$$

↓

distance plane flies.

Work for problem 3(b)

Acceleration of the plane equals zero where the graph changes concavity. There are 2 such instances one at $t = 10$ min & the other $t \in (25, 30)$.

Continue problem 3 on page 9.

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Work for problem 3(c)

$$a(t) = \frac{df}{dt} = \frac{-1}{10}\sin\left(\frac{t}{10}\right) + \frac{21}{40}\cos\left(\frac{7t}{40}\right)$$

$$a(23) = \frac{-1}{10}\sin(2.3) + \frac{21}{40}\cos\left(\frac{161}{40}\right) \approx -0.408 \text{ miles}/\text{min}^2.$$

Work for problem 3(d)

$$\text{avg velocity} = \frac{1}{40-0}\int_0^{40} f(t)\,dt = \frac{1}{40}\int_0^{40} 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$$

$$= \frac{1}{40}\left[6t + 10\sin\left(\frac{t}{10}\right) - 3\cos\left(\frac{7t}{40}\right)\left(\frac{40}{7}\right)\right]_0^{40}$$

$$= \frac{1}{40}\left[6t + 10\sin\left(\frac{t}{10}\right) - \frac{120}{7}\cos\left(\frac{7t}{40}\right)\right]_0^{40}$$

$$= \frac{1}{40}\left[240 - 7.568 - 12.924 - \left(-\frac{120}{7}\right)\right]$$

$$= 5.916 \ \frac{\text{miles}}{\text{min}}$$

END OF PART A OF SECTION II

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CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

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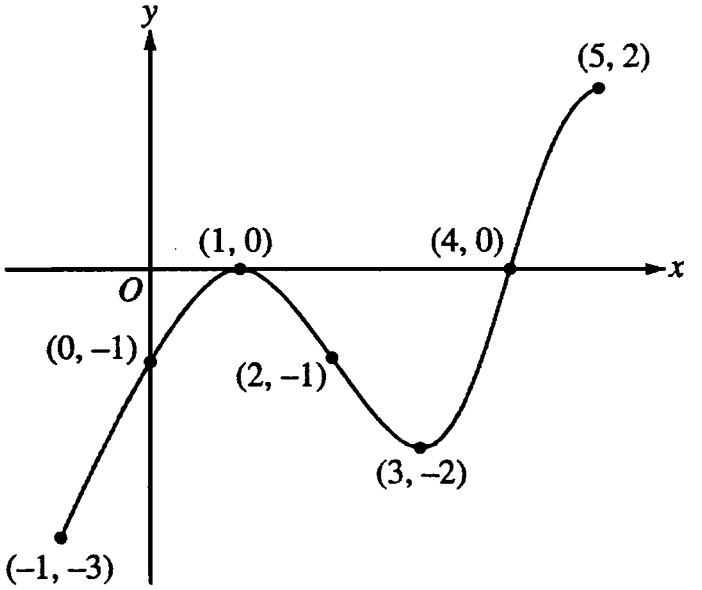

Graph of f'

Work for problem 4(a)

The two points of inflections of f are at $x=1$ and $x=3$.

reason: $f''(x) > 0$ for $x \in (-1, 1)$

$f''(x) < 0$ for $x \in (1, 3)$

$f''(x) > 0$ for $x \in (3, 5)$

Continue problem 4 on page 11.

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A2

NO CALCULATOR ALLOWED

Work for problem 4(b)

f has an absolute minimum at x = 4

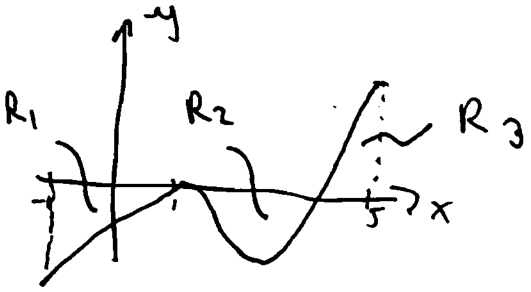
reason : $f'(4) = 0$ and :

x	-1	1	4	5
f'	−	−	+	
f	↘	↘	↗	

↳ abs. min.

f has an absolute maximum at x = −1

reason :



$R_1 + R_2 > R_3 \Rightarrow$

$f(-1) > f(5)$

Work for problem 4(c)

$g(x) = x\,f(x)$

$g'(x) = (x)'\,f(x) + x\,[f'(x)]$

$g'(2) = f(2) + 2f'(2)$

$= 6 + 2\cdot(-1)$

$= 4$

$g(2) = 2\cdot f(2)$

$= 12$

$(2, 12)$

$4 = \frac{y-12}{x-2}$

$4x - 8 + 12 = y$

$y = 4x + 4$

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C_1

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Number of problems—3

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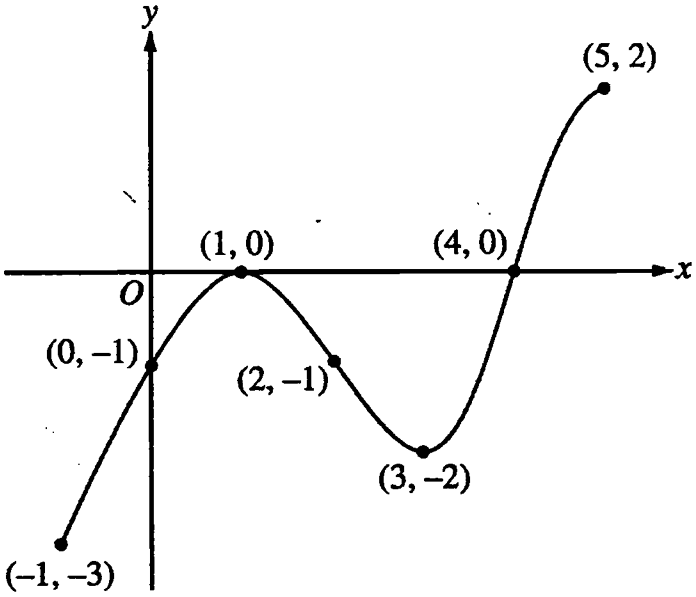

Graph of f'

Work for problem 4(a)

Inflection ⇒ $f''(x)$ changes sign, $f''(x)=0$

⇒ slope of $f'(x)$ changes sign f' slope $= 0$

at $\boxed{x=1}$ slope of $f'(x)$ from +ve to –ve ⇒ inflection

at $\boxed{x=3}$ slope of $f'(x)$ from –ve to +ve ⇒ inflection

Continue problem 4 on page 11.

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C2

Work for problem 4(b)

minimum ⇒ $f'(x)=0$ and $f'(x)$ changes from -ve to +ve

$f'(x)=0 \Rightarrow x=4$

x	-1	4	5
$f'(x)$	$-$		$+$

⇒ local minimum at $\underline{\underline{x=4}}$ and absolute minimum

maximum ⇒ $f'(x)$ and $f'(x)$ changes from +ve to -ve
but there is no such pt ⇒ check endpoints

The decrease from $x=-1$ to $x=4$ is more than increase from $x=4$ to $x=5$

$\Rightarrow f(5) < f(-1)$ ⇒ abs. max at $\underline{\underline{x=5}}$

Work for problem 4(c)

$g(x)=xf(x)$

$g'(x)=f(x)+xf'(x)$

$g'(2)=f(2)+2f'(2)= 6+2(-1)=4$

$\Rightarrow \quad g(2)=2f(2)=2(6)=12$

$\Rightarrow \quad y-12=4(x-2)$

eq of tan $\underline{\underline{y=4x+4}}$

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B1

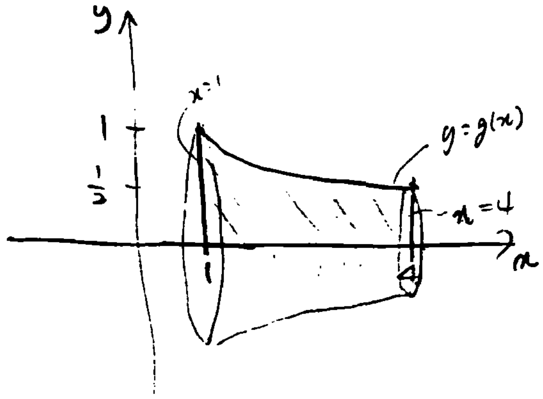
NO CALCULATOR ALLOWED

Work for problem 5(a)

average value of g : $\frac{\int_1^4 g(x)\,dx}{3} = \frac{\int_1^4 x^{-\frac{1}{2}}\,dx}{3} = \frac{\left[2x^{\frac{1}{2}}\right]_1^4}{3} = \frac{4-2}{3}$

$= \boxed{\frac{2}{3}}$

Work for problem 5(b)



$\int_1^4 (g(x))^2 \cdot \pi\,dx$

$= \int_1^4 \frac{1}{x} \cdot \pi\,dx$

$= \left[\pi \ln x\right]_1^4 = \pi \ln 4 - \pi \ln 1 = \boxed{\pi \ln 4}$

Continue problem 5 on page 13.

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B2

NO CALCULATOR ALLOWED

Work for problem 5(c)

average value of the areas $= \dfrac{\text{volume for } x=1 \text{ to } x=4}{3}$

$$= \boxed{\frac{\pi \ln 4}{3}}$$

Work for problem 5(d)

$$\int_4^\infty g(x)\,dx = \lim_{n\to\infty}\int_4^n \frac{1}{\sqrt{x}}\,dx = \lim_{n\to\infty}\int_4^n x^{-\frac{1}{2}}\,dx = \lim_{n\to\infty}\left[2x^{\frac{1}{2}}\right]_4^n$$

$$= \lim_{n\to\infty} 2n^{\frac{1}{2}} - 4 = \underline{\infty}_{//} \rightarrow \text{divergent}$$

average value of g on the interval $[4,\infty)$

$$: \lim_{b\to\infty}\left[\frac{\int_4^b g(x)\,dx}{b-4}\right] = \lim_{b\to\infty}\frac{2b^{\frac{1}{2}}-4^{\to\infty}}{b-4_{\to\infty}} \overset{/}{=} \lim_{b\to\infty}\frac{2\cdot\frac{1}{2}b^{-\frac{1}{2}}}{1} = \lim_{b\to\infty}\frac{1}{\sqrt{b}} = \underline{0}_{//} \rightarrow \text{finite}$$

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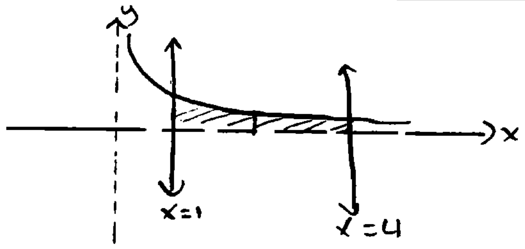
D1

Work for problem 5(a)

$$\text{Avg } g(x) = \frac{1}{4-1}\int_1^4 \frac{1}{\sqrt{x}}dx = \frac{1}{3}\int_1^4 x^{-1/2}dx = \frac{1}{3}\left[2\sqrt{x}\right]_1^4 = \frac{1}{3}\left[2\sqrt{4}-2\sqrt{1}\right]$$

$$= \frac{1}{3}\left[2(2)-2\right] = \frac{2}{3}$$

Work for problem 5(b)



$$V = \pi\int_1^4 R^2(x) - r^2(x)\, dx \qquad R(x) = \frac{1}{\sqrt{x}} \Rightarrow R^2(x) = \frac{1}{x}$$

$$r(x) = 0$$

$$= \pi\int_1^4 \frac{1}{x}dx$$

$$= \pi\left[\ln x\right]_1^4 = \pi \ln 4 \text{ units}^3$$

Continue problem 5 on page 13.

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A_2

Work for problem 5(c)

length of cross sections perpendicular to x-axis $= \frac{1}{\sqrt{x}}$ $\quad A = \left(\frac{1}{\sqrt{x}}\right)^2 = \frac{1}{x}$

$$\text{avg} = \frac{1}{4-1}\int_1^4 \left(\frac{1}{\sqrt{x}}\right)^2 dx = \frac{1}{3}\int_1^4 \frac{1}{x}\,dx = \frac{1}{3}\ln x\Big]_1^4 = \frac{1}{3}\ln 4$$

Work for problem 5(d)

$$\int_4^\infty g(x)\,dx = \lim_{a\to\infty}\int_4^a \frac{1}{\sqrt{x}}\,dx = \lim_{a\to\infty} 2\sqrt{x}\Big]_4^a = \lim_{a\to\infty} 2\sqrt{a} - 2\sqrt{4} = \infty$$

$\Rightarrow$ the improper integral $\int_4^\infty g(x)\,dx$ diverges.

$$\lim_{b\to\infty}\frac{\int_a^b f(x)\,dx}{b-a} \Rightarrow \text{average value of } g = \lim_{b\to\infty}\frac{\int_a^b g(x)\,dx}{b-a}$$

$$= \lim_{b\to\infty}\frac{\int_4^b g(x)\,dx}{b-4}$$

$$= \lim_{b\to\infty} g(x) = 0$$

$\Rightarrow$ avg value of g on $[4,\infty)$ is finite.

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AP® Calculus BC
2004 Sample Student Responses
Form B

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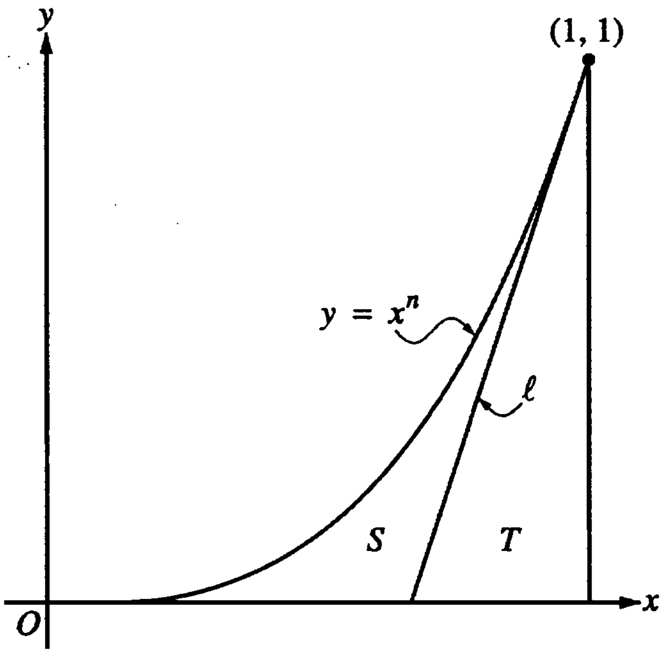

Work for problem 6(a)

$$\int_0^1 x^n\,dx = \left(\frac{x^{n+1}}{n+1}\right)_0^1 = \frac{1^{n+1}}{n+1} - \cancel{\frac{\overset{0}{0^{n+1}}}{n+1}}$$

$$= \frac{1}{n+1}$$

Work for problem 6(b)

$y = x^n$

$y' = nx^{n-1}$

$y'(1) = n\,1^{n-1}$

$= n$

i) the equation of the tangent line will be

$y-1 = n(x-1)$

$\boxed{y = nx-n+1}$

→ in order to get base of T,

$y = nx-n+1$

$0 = nx-n+1$

$\frac{n-1}{n} = x$

$A_{of\,T} = \frac{1 \times \left(1-\frac{n-1}{n}\right)}{2}$

$= \frac{\frac{n-n+1}{n}}{2}$

$\boxed{= \frac{1}{2n}}$

Continue problem 6 on page 15.

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B_2

Work for problem 6(c)

$$A_S = \int_0^1 x^n\,dx - A_T$$

from (a) & b) we know $\int_0^1 x^n\,dx$ & A_T

$$\therefore A_S = \frac{1}{n+1} - \frac{1}{2n}$$

$$= \frac{2n-n-1}{2n(n+1)} = \boxed{\frac{+n-1}{2n(n+1)}}$$

~~Answer~~

$$A_S = \frac{(n-1)}{2n^2+2n}$$

$$A'_S = \frac{2n^2+2n-(n-1)(4n+2)}{(2n^2+2n)^2} = 0$$

$$2n^2+2n-(4n^2-2n-2) = 0$$

$$n^2+n-2n^2+n+1 = 0$$

$$-n^2+2n+1 = 0.$$

by quadratic formula.

$$\frac{-2\pm\sqrt{4+4}}{-2} = \frac{-2\pm 2\sqrt{2}}{-2} = 1\pm\sqrt{2}$$

$$\to A_s(1+\sqrt{2}) = \frac{\sqrt{2}}{2(1+2\sqrt{2}+2)+2+2\sqrt{2}}$$

$$= \frac{\sqrt{2}}{\cancel{6}+2+2\sqrt{2}+4\sqrt{2}}$$

$$= \frac{\sqrt{2}}{8+6\sqrt{2}}$$

$$A_S(1-\sqrt{2}) = \frac{-\sqrt{2}}{2(1-2\sqrt{2}+2)+2(1-\sqrt{2})}$$

$$= \frac{-\sqrt{2}}{\cancel{6}-2\sqrt{2}+2-2\sqrt{2}}$$

$$= \cancel{\frac{-\sqrt{2}}{8-2\sqrt{2}}}$$

~~Answer~~

$$\boxed{n = 1+\sqrt{2}}$$

END OF EXAMINATION

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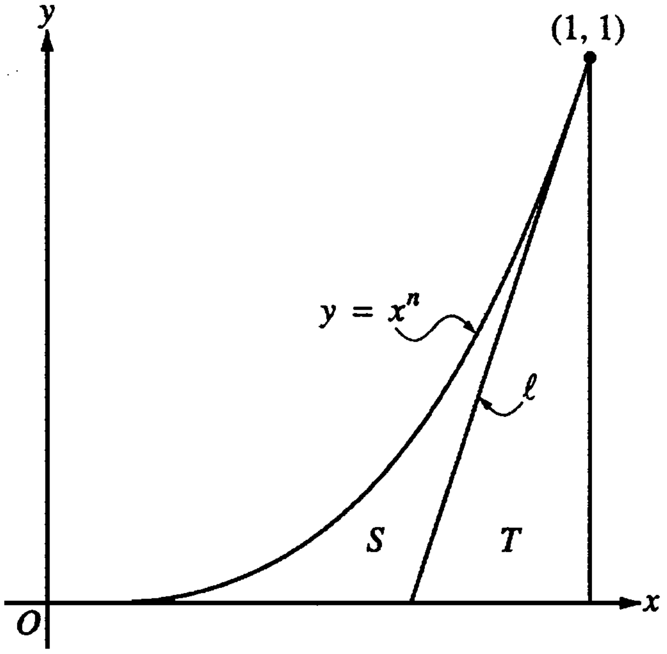

Work for problem 6(a)

$\int_0^1 x^n \, dx = \frac{x^{n+1}}{n+1}\Big]_0^1 = \left(\frac{(1)^{n+1}}{n+1}\right) - \left(\frac{(0)^{n+1}}{n+1}\right)$

$= \frac{1}{(n+1)} - \frac{0}{n+1} = \frac{1}{n+1}$ units^2

Work for problem 6(b)

$y = x^n$

$\frac{dy}{dx} = nx^{n-1}$

$\frac{dy}{dx}\Big|_1 = n$

eqn. of line ℓ:

~~$y - 1 =$~~ $y - y_0 = m(x - x_0)$

$y - 1 = n(x - 1)$

$y - 1 = nx - n$

$y = nx - n + 1$

x-intercept ⇒ ~~$y = 0 \quad = \quad -n + 1 = 0$~~

$y = 0$ ⇒ ~~$n(x-1) + 1 = 0$~~

~~$n =$~~

$nx - n + 1 = 0$

$nx = n - 1$

$x = \frac{n-1}{n}$

Area $= \frac{1}{2}(1)\left(1 - \frac{n-1}{n}\right)$

Area $= \frac{1}{2}(1)\left(|\frac{n-n-1}{n}|\right) = \frac{1}{2}(1)(1 - \frac{1}{n}|) = \frac{1}{2}(1)(\frac{1}{n}| = \frac{1}{2n}$ units^2

Continue problem 6 on page 15.

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D$_2$

NO CALCULATOR ALLOWED

Work for problem 6(c)

$$\text{Area of } S = \int_0^1 x^n\,dx - \text{Area of } T$$

$$= \frac{1}{n+1} - \frac{1}{2n}$$

$$= \frac{(2n) - (n+1)}{2n(n+1)}$$

~~= n+1~~ $\frac{n-1}{2n^2+2n}$ units^2

maximum : $S'(n) = 0$

$\Rightarrow$ $(1)(2n^2+2n) - (4n+2)(n-1) = 0$

$2n^2+2n - (4n^2 - 4n + 2n - 2) = 0$

$2n^2 + \cancel{2}n - 4n^2 + 4n - \cancel{2}n - 2 = 0$

$-2n^2 + 4n - 2 = 0$

$n^2 - 2n + 1 = 0$

$(n-1)^2 = 0 \Rightarrow n = 1$ will maximize the area.

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